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ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the fourth grade mathematics course. Contents include: (1) Overview of Grade 4 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, instructional strategies, and uses of technology and manipulatives); (2) Essential Elements of Instruction with sample learning objectives and sample clarifying activities; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, and targets); (4) Sample Lessons for Teaching Grade 4 Mathematics; and (5) Evaluation (philosophy and types of evaluation). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 references. (MKR)

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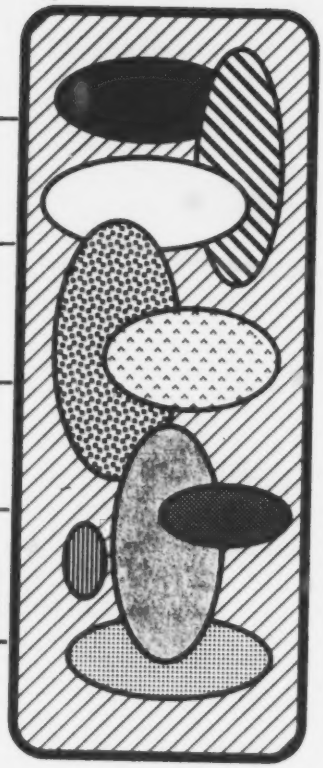

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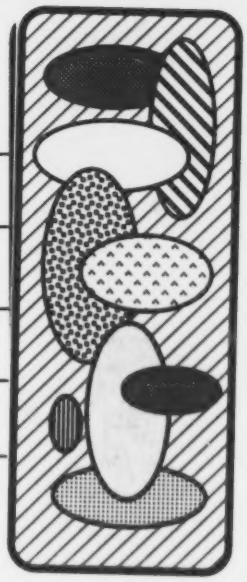

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FOREWORD

Guidelines for Teaching Grade 4 Mathematics is designed to help teachers and other school district personnel plan and teach fourth grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for fourth grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 4 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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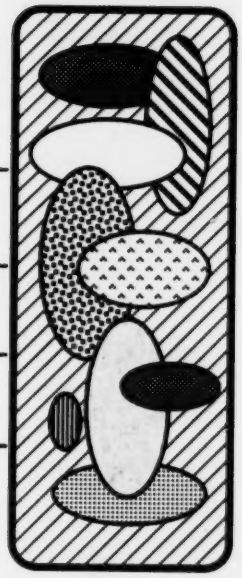
Overview of Grade 4 Mathematics



Mission Statement

Guidelines for Teaching Grade 4 Mathematics is one in a series of eight documents for the first through the eighth grades designed to assist teachers and other school personnel in the planning and teaching of elementary mathematics. The discussions of philosophy, goals, instructional strategies, uses of technology and manipulatives, and aspects of evaluation are provided as starting points for districts to begin the process of developing their own curriculum documents. The essential elements of instruction for each grade level are supported with sample learning objectives, sample clarifying activities, and complete sample lessons. These guidelines should prove useful to district personnel in: (1) planning curriculum, (2) planning instruction, and (3) equipping classrooms for mathematics teaching and learning.

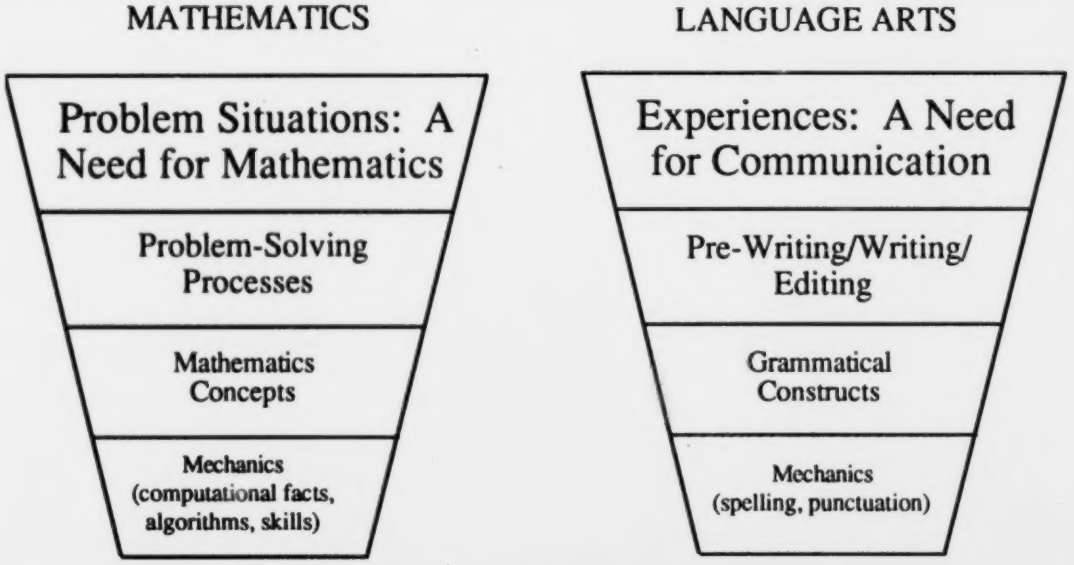
Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all elementary school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through rich, worthwhile mathematical experiences. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in the elementary grades should be broad-based and concept driven and should reflect relevant mathematics and connections between mathematics concepts and between these concepts and other disciplines.

Children enter elementary school with a natural curiosity and enthusiasm for learning. Mathematics experiences at the elementary level should tap into these characteristics for children to begin developing mathematical power—the ability to think and communicate, drawing on mathematical ideas and using mathematical tools and techniques. The attitudes students form in

elementary school toward mathematics will determine the choices they make of future mathematics coursework and consequently the availability or loss of educational and career opportunities.

The elementary school mathematics curriculum should emphasize the processes of problem solving, reasoning, communication, and making connections within the contexts of investigating geometry, measurement, probability, statistics, graphing, patterns, and functions, as well as with number, numeration, and operation concepts. Problem solving should be the focus of instruction with skills and concepts being introduced, developed, and applied through meaningful problem situations. Mathematics instruction needs to begin with meaning and purpose in much the same way as elementary teachers present language arts instruction, as reflected in the following graphic illustration:



All students need rich and relevant problem-solving experiences with appropriate teacher guidance and questioning. Such experiences will empower students to build meaning for the mathematics they encounter today and to strengthen reasoning skills needed for the mathematics of tomorrow.

Goals

According to *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council fo Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability
- becoming mathematical problem solvers
- learning to communicate mathematically
- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to elementary school students must be related to the characteristics of the learners and their needs today and in the future.

Everybody Counts (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (p. 45). Individuals must be able to use mathematics in their later lives—as employees, parents, and citizens. Ability and disposition to do so often depends on attitudes toward mathematics developed in school.

Through the use of worthwhile mathematical activities investigated in cooperative, group environments, teachers of elementary mathematics can empower their students with strong mathematical understanding and disposition.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. The standards describe the nature of the tasks, patterns of communication, the learning environment, and the analysis of instruction. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;

- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;

- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

STANDARD 6: ANALYSIS OF TEACHING AND LEARNING

The teacher of mathematics should engage in ongoing analysis of teaching and learning by:

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;

in order to:

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- make plans both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

The movement toward this vision of instruction for mathematical empowerment of all students is strongly dependent upon the environment of the classroom, an environment governed in a large part by the decision-making role of the classroom teacher. The NCTM teaching standards identify five major components necessary in the instructional environment for the mathematics classroom and tie these components directly to teachers asking, and encouraging students to ask, appropriate and stimulating questions. The five major instructional components and suggestions for questions are (NCTM, 1991, pp. 3-4):

- **Helping students work together to make sense of mathematics**
  "What do others think about what Janine said?"
  "Do you agree? Disagree?"
  "Does anyone have the same answer but a different way to explain it?"
  "Would you ask the rest of the class that question?"
  "Do you understand what they are saying?"
  "Can you convince the rest of us that that makes sense?"

- **Helping students to rely more on themselves to determine whether something is mathematically correct**
  "Why do you think that?"
  "Why is that true?"
  "How did you reach that conclusion?"
  "Does that make sense?"
  "Can you make a model to show that?"

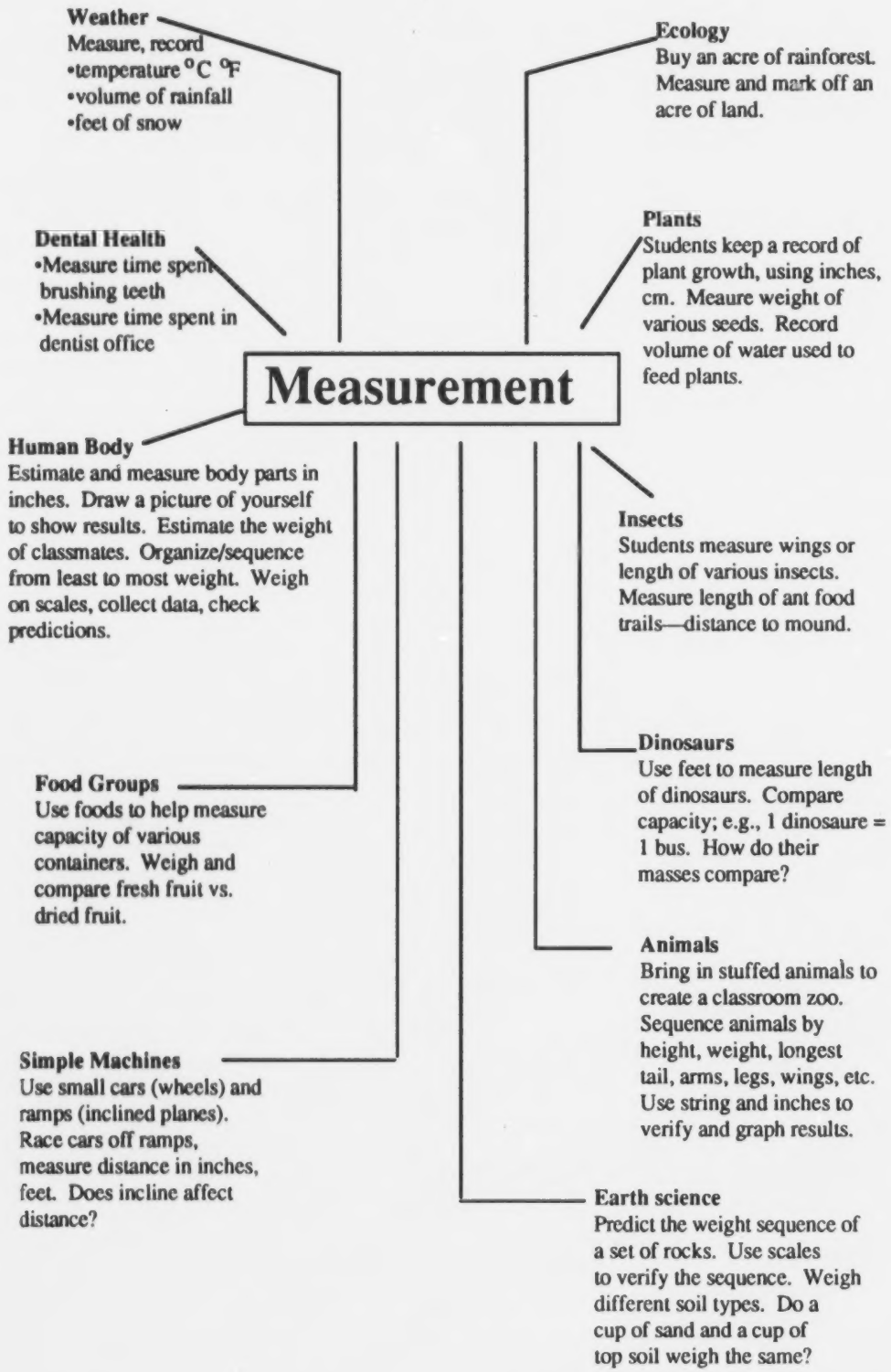
- **Helping students learn to reason mathematically**
  "Does that always work?"
  "Is that true for all cases?"
  "Can you think of a counterexample?"
  "How could you prove that?"
  "What assumptions are you making?"

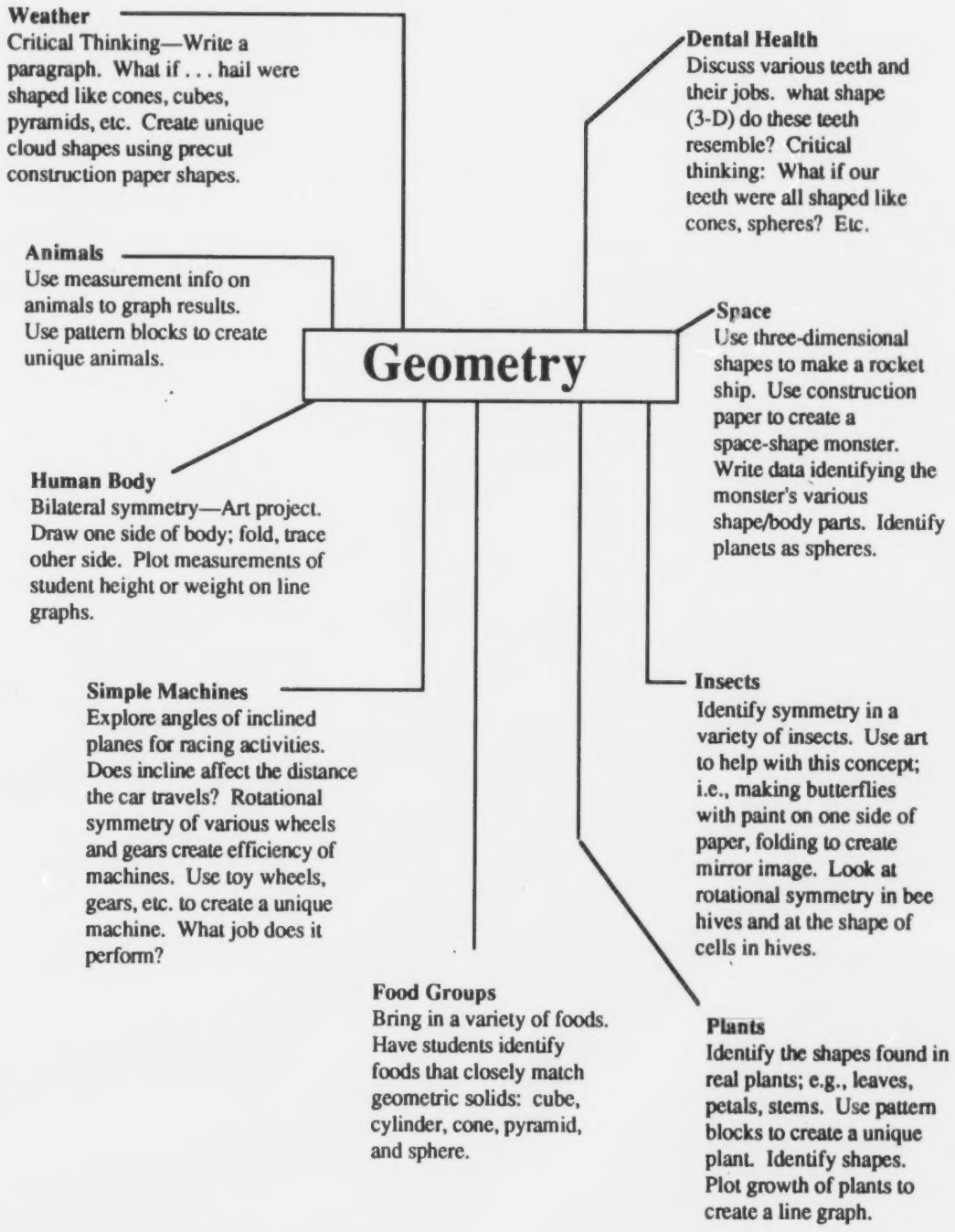
- **Helping students learn to conjecture, invent, and solve problems**
  "What would happen if . . .." What if not?"
  "Do you see a pattern?"
  "What are some possibilities here?"
  "Can you predict the next one? What about the last one?"
  "How did you think about the problem?
  "What decision do you think he should make?"
  "What is alike and what is different about your method of solution and hers?"

- **Helping students to connect mathematics, its ideas, and its applications**
  "How does this relate to . . .?"
  "What ideas that we have learned before were useful in solving this problem?"
  "Have we ever solved a problem like this one before?"
  "What uses of mathematics did you find in the newspaper last night?"
  "Can you give me an example of . . .?"

Instructional Strategies

The following diagrams are examples of one teacher's planning efforts to connect measurement and geometry concepts to life, earth, and physical science units:


Weather
Measure, record
•temperature °C °F
•volume of rainfall
•feet of snow
Ecology
Buy an acre of rainforest. Measure and mark off an acre of land.
Dental Health
•Measure time spent brushing teeth
•Measure time spent in dentist office
Plants
Students keep a record of plant growth, using inches, cm. Meaure weight of various seeds. Record volume of water used to feed plants.
Measurement
Human Body
Estimate and measure body parts in inches. Draw a picture of yourself to show results. Estimate the weight of classmates. Organize/sequence from least to most weight. Weigh on scales, collect data, check predictions.
Insects
Students measure wings or length of various insects. Measure length of ant food trails—distance to mound.
Dinosaurs
Use feet to measure length of dinosaurs. Compare capacity; e.g., 1 dinosaure = 1 bus. How do their masses compare?
Food Groups
Use foods to help measure capacity of various containers. Weigh and compare fresh fruit vs. dried fruit.
Animals
Bring in stuffed animals to create a classroom zoo. Sequence animals by height, weight, longest tail, arms, legs, wings, etc. Use string and inches to verify and graph results.
Simple Machines
Use small cars (wheels) and ramps (inclined planes). Race cars off ramps, measure distance in inches, feet. Does incline affect distance?
Earth science
Predict the weight sequence of a set of rocks. Use scales to verify the sequence. Weigh different soil types. Do a cup of sand and a cup of top soil weigh the same?


Geometry
Weather
Critical Thinking—Write a paragraph. What if . . . hail were shaped like cones, cubes, pyramids, etc. Create unique cloud shapes using precut construction paper shapes.
Animals
Use measurement info on animals to graph results. Use pattern blocks to create unique animals.
Human Body
Bilateral symmetry—Art project. Draw one side of body; fold, trace other side. Plot measurements of student height or weight on line graphs.
Simple Machines
Explore angles of inclined planes for racing activities. Does incline affect the distance the car travels? Rotational symmetry of various wheels and gears create efficiency of machines. Use toy wheels, gears, etc. to create a unique machine. What job does it perform?
Food Groups
Bring in a variety of foods. Have students identify foods that closely match geometric solids: cube, cylinder, cone, pyramid, and sphere.
Plants
Identify the shapes found in real plants; e.g., leaves, petals, stems. Use pattern blocks to create a unique plant. Identify shapes. Plot growth of plants to create a line graph.
Insects
Identify symmetry in a variety of insects. Use art to help with this concept; i.e., making butterflies with paint on one side of paper, folding to create mirror image. Look at rotational symmetry in bee hives and at the shape of cells in hives.
Space
Use three-dimensional shapes to make a rocket ship. Use construction paper to create a space-shape monster. Write data identifying the monster's various shape/body parts. Identify planets as spheres.
Dental Health
Discuss various teeth and their jobs. what shape (3-D) do these teeth resemble? Critical thinking: What if our teeth were all shaped like cones, spheres? Etc.

Appropriate questioning techniques and meaningful problem-solving situations are two major strategies for effective mathematics instruction.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The NCTM *Curriculum Standards* (1989) call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students an important learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *Ed Talk: What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative; they need to experiment and share ideas.

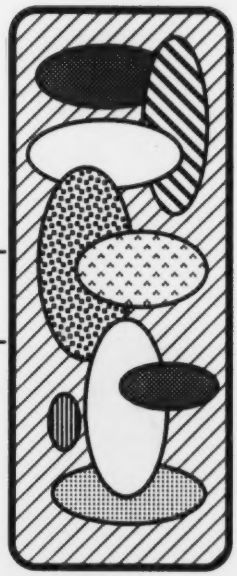
Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fraction independent of the physical representation. In addition to using manipulatives for new concepts, activities

should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

However, the use of manipulatives should not become an end in itself. Learning the motions of modeling addition and subtraction with Cuisenaire rods does not guarantee understanding of the mathematical relationship between these inverse operations. It is important not only that the concrete manipulation of materials closely matches the mathematical concept being developed but that the actions are accompanied with appropriate questioning by the teacher and reflection by the student.

In the instructional uses of both technology and manipulatives, the goal is to enhance mathematical thinking. Again, the teacher's role as questioner and decision maker influences the effectiveness of the incorporation of these tools.

Essential Elements of Instruction



Essential Elements of Grade 4 Mathematics with Sample Learning Objectives and Sample Clarifying Activities

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The following essential elements and descriptors for fourth grade mathematics are annotated with sample learning objectives and sample clarifying activities (except for EE1: Problem-Solving). The learning objectives give a brief look at the developmental components of the concept(s) in the descriptor. The sample clarifying activities are addressed to the teacher and provide a glimpse of what student engagement with this concept might look like in the classroom.

Each set of sample learning objectives and sample clarifying activities is meant to be viewed as an integrated whole (not necessarily matched one-to-one) to clarify the descriptors and to identify connections among them, as well as connections to meaningful problem situations. The Problem-Solving strand therefore is annotated only with sample learning objectives and is connected to the other strands through the language and situations used in their sample clarifying activities.

Many of the activities involve the use of manipulatives and common materials such as hundreds charts or grid paper. A list of these manipulatives can be found in the References and Resources. Also in the References and Resources are lists of the children's trade books, teacher resource books, and software cited in the activities as examples of instructional materials.

The revised essential elements, sample learning objectives, and sample clarifying activities for Grade 4 are:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 4;

Sample Learning Objective

Involving patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; probability, statistics, and graphing

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

Sample Learning Objective

Make inferences and predictions

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

Sample Learning Objectives

Demonstrate creative thinking through fluency, flexibility, elaboration, creation of new ideas, and spontaneity

Estimate outcomes including appropriate units for outcomes

(D) select appropriate strategies from a variety of approaches;

Sample Learning Objective

Such as: acting it out; making a model; drawing a picture; guessing and checking; making a diagram; chart, or graph; finding a pattern; using a simpler problem; working backwards; etc.

(E) select appropriate materials and methods for solutions; and

Sample Learning Objectives

Such as: concrete manipulatives, mental computation, paper and pencil (pictorial and/or symbolic), calculator, or computer

Reflect on the problem-solving process and solution of a problem by evaluating outcomes for reasonableness (including appropriateness of units), make revisions as needed, describe and discuss the process and solution, and make a decision based on the solution

(F) generate and extend problems.

(2) Patterns, Relations, and Functions. Use of models and patterns to develop the concepts of relations and functions. The student shall be provided opportunities to:

(A) investigate patterns on the multiplication table;

Sample Clarifying Activities

Have students color in all the even products on a multiplication facts table. Ask questions such as, "What pattern do you see? Why do you think that pattern is formed? Write a description of what you think causes a product to be even. Do the same thing for odd products."

Have students work in pairs to guess how many products in the table are over 50 and check their guesses using the table.

Have students work in small groups to address questions such as, "What pattern do you see in the 9's row and column? What other patterns do you see in any other rows or columns? Write an argument for why you think a certain pattern occurs."

Have students make a graph of how often each number appears as a product on the multiplication table and look for patterns. Students can write a possible explanation for the pattern or for why there is no obvious pattern.

(B) record the pairing of members of two sets, determine the relationship between each pair, and use the rule to generate additional ordered pairs, using a calculator or computer when appropriate;

Sample Learning Objectives

Identify two related sets of information.

Make a chart showing each element of one set paired with a related element from the other set.

Organize the pairs in the chart to illustrate the pattern, or relationship, between the sets of data.

Use the identified pattern (relationship, rule) to generate new ordered pairs to extend the chart and make predictions.

Sample Clarifying Activities

Have students investigate a pattern using the constant function on a calculator and record the data as ordered pairs in a chart. (See example chart.)

Number of times constant key is pushed	Number display on calculator
0	100
1	98
2	96
3	94
4	92
5	90
6	88
7	86
8	84
9	82
10	80

Students can describe the pattern, predict what would come next, and use the calculator to test predictions.

Have students play Blockers and Finders (Sunburst Communications) with a partner. Partners can write an explanation for one of the strategies used to locate a blocker and to determine what kind it is.

(C) explore patterns of problems with inverses in computation using a calculator or computer when appropriate.

Sample Learning Objectives	*Sample Clarifying Activities*
Explore the inverse relationship between addition and subtraction.	Have students use calculators to solve number riddles. Example: "If you add 54 to me, you get 72. What am I?" Students can discuss their strategies for choosing which operation to use on the calculator and write some riddles of their own to share with other students.
Explore the inverse relationship between multiplication and division.	Show students a picture of grouped objects and have them write multiplication stories and division stories that could go with the picture. Students can describe the differences between the stories.

(3) Number and Numeration Concepts. Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) use concrete materials to represent factor pairs of numbers;

Sample Clarifying Activities

Have students use chips or tiles to build arrays to show all the factors of a number.

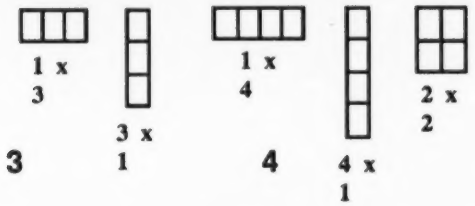
Have students demonstrate array patterns on geoboards for all the factors of a number.

Have students model multiplication with "ice cream cones" for the factors of a given product. Example: Twelve could be modeled by 6 different kinds of ice cream on two different kinds of cones for 12 different ice cream cones.

(B) use the patterns of factor pairs to identify prime and composite numbers;

Sample Clarifying Activities

Have students use chips or tiles to build all array patterns that represent each number from 1 through 25 and investigate the number of arrays that result. Example:



Students can write true statements about the arrays. Examples: Even numbers above 2 have at least 3 arrays. The numbers 1, 4, 9, 16, and 25 have square arrays. Some numbers have only 2 arrays. Students can use these observations to develop definitions of *prime* and *composite*. (See *Factors and Multiples*. developed by the Middle Grades Mathematics Project)

Have students number off (1-22) and stand in order around the room. Begin by having all the numbers divisible by 2 (except 2) sit down, then by 3 (except 3), by 5, etc. until only prime numbers are left standing. Discuss the numbers remaining and the class definition of *prime* in the preceding activity. Do the numbers remaining fit the class definition? Why or why not? Students can do research on the Sieve of Eratosthenes.

(C) use concrete models to generate patterns of equivalent fractions and record the patterns;

Sample Learning Objectives

Use concrete models to demonstrate equivalent fractions.

Sample Clarifying Activities

Have students use pattern blocks to represent fractions. Let the yellow hexagon represent *one*. Ask questions such as, "Which arrangement(s) of single color blocks represent 1/2? 1/3? What if 2 hexagons = 1? Which arrangement(s) of single color blocks now represent 1/2? 1/3? 1/4? 1/6?" Students can record in a table and look for patterns.

Identify patterns and make generalizations about how to record equivalent fractions.

Have students fold a sheet of paper and shade one side or one half of the paper, then fold this again in half so that the paper is in fourths. Students can continue folding the paper into eighths and sixteenths and record the equivalent fractions represented by the folds in the paper.

Identify equivalent fractions by simplifying them to lowest terms.

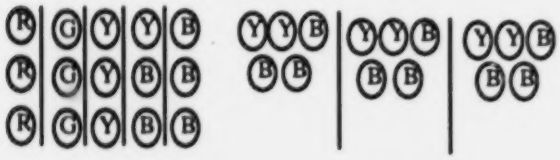
Have students use Cuisenaire Rods to build a "wall" where the orange + red rods = 1.

<table>
<tr><td colspan="11">orange</td><td>red</td></tr>
<tr><td colspan="6">dark green</td><td colspan="6">dark green</td></tr>
<tr><td colspan="4">purple</td><td colspan="4">purple</td><td colspan="4">purple</td></tr>
<tr><td colspan="3">light green</td><td colspan="3">light green</td><td colspan="3">light green</td><td colspan="3">light green</td></tr>
<tr><td colspan="2">red</td><td colspan="2">red</td><td colspan="2">red</td><td colspan="2">red</td><td colspan="2">red</td><td colspan="2">red</td></tr>
<tr><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td></tr>
</table>

Students can use the "wall" to identify equivalent fractions. Example: 1 dark green (1/2) = 2 light greens (2/4), 3 reds (3/6)=6 whites (6/12).

On cm graph paper, have students outline a 4x6 array and use the array to identify equivalent fractions. Example: How many squares are in 1/2? 1/3? 1/4? 1/6? Students can illustrate and record as many names as possible for each fractional part of the array. Example: 1/2 = 12/24 = 6/12 = 4/8.

Have students use edible manipulatives (like M&M's or Skittles) to model fractional parts of a set. Students can record fractional observations in a variety of ways using equivalent fractions.
Example:
15 M&M's: 3 red, 2 green, 5 yellow, 5 brown



3/15 = 1/5 are red

5/15 = 1/3 are not brown or yellow

Students can use a Math Explorer calculator to generate equivalent fractions by entering a decimal such as 0.25, changing it to fractional form with the F-D key, and continuing to simplify it until it is in its simplest form. Have students record the equivalent fractions and the factors used throughout the process.

Have students use counters and concepts of multiples to build equivalent fractions. Students can record the equivalent fractions in a chart and describe the patterns. Have students choose one additional manipulative (graph paper, tiles, Cuisenaire Rods, fraction bars, pattern blocks, etc.) to represent the equivalent fractions in the chart.

(D) use physical models to represent mixed numbers;

Sample Clarifying Activities

Have students use pattern blocks or Cuisenaire Rods to represent mixed numbers. Example: If 1 yellow hexagon = 1, what collection of pattern blocks could = 1 and 1/2? 2 and 2/3? 3 and 1/6?

Use a 10x10 grid to represent 1. Ask questions such as, "What collection of base ten blocks would represent 4 and 3/10? 3 and 1/2? What mixed number would 2 grids, 2 orange rods, and 5 white rods represent?"

Students can use Math Explorer calculators to explore mixed numbers by entering an improper fraction like 45/12 and using the "Ab/c" key to change it to a mixed number and then the "x-y" key to change it back to an improper fraction. Have students explore a variety of improper fractions. Does the calculator always change the mixed number back to the original improper fraction? [See TEA Module 20, Grades 3-6, Calculator.]

(E) use concrete models to demonstrate place value concepts of decimals;

Sample Learning Objectives	*Sample Clarifying Activities*
Use money to extend place value concepts to tenths and hundredths.	Have students use dollars, dimes, and pennies to represent ones, tenths, and hundredths.

Use concrete models (10 x 10 grid) to develop place value concepts to tenths and hundredths.

Have students use base ten blocks to represent decimals. Example: If the 10x10 flat = 1, represent 2.4, 1.8, 3.35. (See TEA Module 5, Grades 3-6, Numeration.)

Identify, read, and write decimals to tenths and hundredths.

Use metric measurement to represent decimals. Example: Measure a collection of items and record in decimal form. Examples: 8cm=.08m, 250ml = .250 L.

(F) use concrete models to compare and order decimals;

Sample Learning Objectives

Sample Clarifying Activities

Use concrete models to compare decimals to tenths and hundredths, and write sentences using <, >, or =.

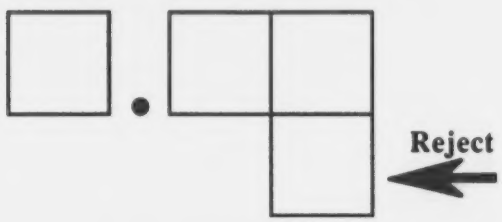
Have students build models of decimals using base ten blocks and develop strategies to compare and order the quantities represented.

Use concrete models to order decimals to tenths and hundredths from greatest to least and least to greatest.

Students can use a collection of 10x10 grids to shade in decimals, cut them out, and order them from smallest to largest.

Have students use food or toy ads from the newspaper to compare decimals. Students can cut out individual items with their prices and order them from most expensive to least expensive.

Have students play Building Decimals in groups of four to six. Each player draws a gameboard like the one below.



Each player takes turns rolling one die and writing the number that comes up in one square of their gameboard. Once a number has been written, it cannot be changed. When all squares are filled, players order their decimals from largest to smallest. Points can be awarded for the largest or smallest decimals. Have students play several rounds. The winner has the most points, can read each of the decimals and justify why they were larger or smaller than other decimals.

Have students bring in the mileage from the odometers on their parents' vehicles and order them from smallest to largest. Students can discuss questions such as, "How old is the car with the largest number? The smallest? Does the oldest car always have the largest number on the odometer?" (These data can also be represented in ordered pairs: (age of vehicle, mileage.)

(G) round whole numbers;

Sample Learning Objectives

Use concrete and pictorial models to round whole numbers to the nearest ten.

Use concrete and pictorial models to round whole numbers to the nearest hundred.

Identify patterns in the results of rounding to devise a strategy for rounding whole numbers.

Sample Clarifying Activities

Have students use grocery store ads to estimate grocery bills and to discuss different strategies used in making the estimates. Students can figure exact costs and see how close they came to their estimates. Repeat the activity using other sources of real-life numbers: catalogues, menus, mileage on a highway map, table of temperatures in a newspaper weather section, team statistics from the sports section, etc. Students can continue to discuss estimation strategies and refine the process to develop a class collection of effective ways to make estimates, record them, and post them in the classroom. They can add to the collection throughout the year.

(4) Operations and Computation. Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) select the appropriate operation and/or strategy to solve a problem and justify the selection;

Sample Learning Objectives

Analyze a problem by acting it out or using manipulatives to demonstrate the actions described in the problem.

Analyze a problem by identifying relationships and discriminating relevant from irrelevant information.

Connect the results of the analysis to the meaning of operations by selecting a strategy that will solve the problem, then justify the selection of that strategy.

Sample Clarifying Activities

Have students work in groups to develop original story problems involving addition, subtraction, multiplication, and/or division. Students can trade with members of another group and solve the new problem by illustrating the action in the problem (using manipulatives, drawings, or by acting it out). Ask questions such as, "Which information do you need to solve the problem? Which information is not necessary? Describe which operation(s) might be used to solve it. Explain why your answer is reasonable."

Have students use information from a baseball card to answer mathematical questions such as "How old was the baseball player when he had his best slugging percentage? Did he get more hits his first three years or his last three years? How would you figure this out?

Have students use information from a chart to solve problems. Example: Tommy Traveler decided to travel around the United States during his one-week vacation. He kept a chart of his trip but never finished it. Use the information given in the questions to complete the chart.

DAY	DRIVING SPEED	HOURS TRAVELED	TOTAL MILES TRAVELED
MON.	45 MPH	5	225
TUES.		7	
WED.	50 MPH		
THURS.		6	300
FRI.	60 MPH		
SAT.			450
SUN.		8	

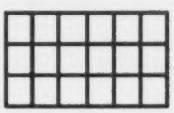
Then use data from the chart to solve the following problems:

1. On Tuesday, the driving speed was 65. What were the total miles traveled?
2. He traveled 376 miles on Sunday. How many miles did he travel on Saturday and Sunday?
3. On Saturday, Tommy drove for nine hours. How fast did he drive that day?
4. Was his driving speed faster on Saturday or Sunday?
5. Tommy was so tired on Friday that he could only drive for two hours. How many miles did he travel?
6. If Tommy traveled eight hours on Wednesday, how many more miles did he travel than he did on Sunday?
7. If, during another week, Tommy traveled eight hours a day, how many hours did he travel?

(B) solve division problems related to multiplication facts;

Sample Clarifying Activities

Have each group choose a different number (such as 18, 20, 24, 42, or 36). Students can select that number of tiles and use them to build all the rectangular arrays possible using all of the tiles. Have students use both multiplication and division sentences to describe each array. Example: The number chosen is 18. One possible array is 3 x 6.



3 x 6 = 18 (3 rows, 6 in each row = 18 tiles)
6 x 3 = 18 (6 columns, 3 in each column = 18 tiles)
18 ÷ 3 = 6 (18 tiles divided into 3 rows = 6 in each row)
18 ÷ 6 = 3 (18 tiles divided into 6 columns = 3 in each column)

Have students invent and describe a variety of situations where you would need to use division/multiplication facts to solve a problem. Example: You have 27 cookies to share with 3 friends and you must decide how many each will get. Or you have 27 cookies and plan to serve 3 cookies to each person. You must decide how many people could share the cookies.

	Have students use real-world objects to generate and solve problems involving sharing or equal groups. Example: There are 64 crayons in a box. If you share them among 8 students, how many will each get? How many tickets will a class of 22 sell if each student sells 7 tickets?

(C) add and subtract decimals with concrete materials and record results;

Sample Learning Objectives	*Sample Clarifying Activities*
Use understanding of the operations of addition and subtraction to respond to real-world problems involving addition and subtraction of decimals.	Have each group use advertising sections from the newspaper to write original problems involving addition/subtraction with decimals. Groups can trade problems and select a manipulative to use to solve another group's problem, then describe the solution strategy and why the manipulative was chosen to solve the problem.
After using manipulatives to solve a problem involving decimals, record the actions and results with decimal symbols.	Have pairs of students use base ten blocks (10x10 grid = 1) to play a game. A game consists of 7 turns for each player. Each player in turn will roll two dice, decide which die will represent tenths and which will represent hundredths, and use the base ten materials to physically subtract the resulting decimal from the 10x10 grid, making all trades as necessary. The winner is the one closest to 0 at the end of 7 turns. A written record should be kept of each turn. After the game, students can analyze each turn to see if a player could have come closer to 0 if the decisions made during the game had been different.
	Have students measure a variety of objects around the room with interlocking centimeter cubes. Students can record the lengths and differences between lengths using decimeters. Example: My math book is 2.3 decimeters tall (23 interlocking cm cubes). My marker is 1.4 decimeters long (14 cm cubes). The difference between the height of my math book and the length of my marker is 0.9 decimeters (9 cm cubes).

(D) explore the volume of a rectangular solid as a geometric model for multiplication of three factors;

Sample Clarifying Activities

Each group uses wooden cubes, sugar cubes, or interlocking cubes to build different rectangular solids. Record on a class chart the three dimensions of each solid and the number of cubes required to build it. Students can organize the data in the chart, look for patterns, and develop a class rule for finding the volume of a rectangular solid.

Have students bring from home a variety of rectangular solids such as tissue boxes, jewelry boxes, and candy or cereal boxes. Using 1" cubes, students can find the volume of each rectangular solid and describe and compare the processes used. Change the unit of measure from 1" cubes to cm cubes. Have students predict the volume of each box using the new unit of measure, record predictions, discuss how they were made, and use cm cubes to find the actual volumes.

(E) recall multiplication facts;

Sample Learning Objectives	*Sample Clarifying Activities*
Use patterns to remember multiplication facts.	Have students brainstorm a long list of real-life rectangular arrays. Examples: eggs in a carton, rows of seats in an auditorium, rows of plants in a garden, tiles in a classroom floor or ceiling. Have each group select one example from the list to illustrate and describe using two different multiplication sentences.
Use the properties of multiplication to remember multiplication facts.	Have students use the distributive property of multiplication over addition to break down a problem into two already known facts. Example: $7x5 = 5x5 + 2x5$. Students can develop a class list of tough facts to remember and make a poster of the tough numbers showing ways they can be broken down to make them easier to remember. Have students work in groups to record the 9's multiplication facts in a variety of ways, look for and describe patterns, and use the patterns to remember the 9's facts.

(F) solve application problems involving multiplication;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify and describe real-world problem situations that can be solved using multiplication.	Have students develop lists of things that come in multiples of 2, 3, 4, . . . 12. (Try to come up with a long list for each number.) Examples: days in a week, legs on an insect, pennies in a nickel, wheels on a tricycle, ears on a dog, players on a baseball team. Post the lists. Students can choose items from the lists to develop into original problems. Example: Angie decided to make thank-you cards to send to the five friends who came to her birthday party. She drew five butterflies on the cards. Each butterfly had 6 legs. How many legs did she draw?
Estimate answers to problems involving multiplication.	
Create and solve application problems involving multiplication.	"The Garden Club bought 24 tulip bulbs to make a garden. The club members immediately got into an argument about how their garden should look. The only thing they could agree on was that it should be rectangular in shape." Have students help the club members settle their disagreement by drawing all the different arrangements in which the tulips can be planted into rectangular gardens. Students can use the story plot to develop an original story involving multiplication.

(G) solve problems involving addition, subtraction, and multiplication of large numbers using calculators;

Sample Learning Objectives	*Sample Clarifying Activities*
Estimate answers to problems with large numbers.	Show students a large container filled with beans. Have students predict the total number of beans in the container, record predictions, and discuss their reasonableness. Each pair of students can create a strategy for finding the exact number of beans. Have students compare strategies and results.
In the context of real-world problems using large numbers, use number sense and understanding of operations to analyze the reasonableness of answers obtained with a calculator.	Bring election results into the classroom and have students analyze them by precinct to determine the winner of each election.
	Do some research and make a timeline of inventions. Determine how much time has elapsed since each invention was created.

Use a Texas highway map to choose two cities such as Amarillo and Longview. Using the mileage recorded for a variety of highways, find the route with the shortest distance between the two cities.

(H) use properties of operations and problem-solving strategies to do mental calculations, extending beyond fact recall.

Sample Learning Objectives	*Sample Clarifying Activities*
Use problem-solving strategies such as recognition of patterns, estimation, guess and check, recognizing relevant information, and number sense to do mental calculations.	Give students three different 3x3 numerical arrays. Have students use only mental mathematics to discover which array is a magic square (each row, column, and diagonal add to the same number). See *Arithmetic Teacher*, December, 1989, for a description of strategies used in creating magic squares.
Use logical reasoning to justify the mental calculation strategies.	Display a checkerboard. Determine how many squares of different sizes are on the board. Develop an organized method and look for patterns. Analyze and compare solution strategies and results.
	Play Secret Number in which each player writes clues about his or her secret number. Example: "If it were a page number in a 100-page book, it would be in the middle half of the book. It is more than 4 tens and less than 6 tens. The tens digit is two less than the ones digit. Together the digits have a sum of 12. What is my secret number?" Analyze the clues. Which numbers would be eliminated by each clue? How would rearranging the clues change the puzzle? Why? Develop a class collection of secret number puzzles.

(5) Measurement. Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) measure areas using grids;

Sample Clarifying Activities

Have students build polygons on a geoboard then count the number of square units enclosed by the rubberband to find the area of each polygon. Students can discuss the attributes of the unit used and justify how it was used to find area.

Have students trace cut-out shapes or objects and choose a unit (circular counters, tiles, pattern-block triangles, etc) to use to find the area of the shape. Students can estimate and determine the area, discuss the attributes of the unit used, and justify how it was used to find area.

Have students trace plant leaves onto grid paper and count squares to determine area as part of a science lesson on photosynthesis. Students can discuss the attributes of the unit used and justify how it was used to find area.

Have students trace a hand or foot onto graph paper, estimate and determine the area. Students can discuss the attributes of the unit used and justify how it was used to find area.

Have students discuss different solution methods for the activities above and develop a class strategy for finding area.

(B) find perimeters using standard and nonstandard units;

Sample Clarifying Activities

Have students estimate how many of their feet it would take to measure the perimeter of the classroom. Students can use cut-outs of their feet to find out. Have students compare results from different class members and discuss why they differ. Students can use a ruler to find the perimeter and compare the results with the feet results. Have students write original stories about a situation in which nonstandard feet are used as a unit of measure.

Have students construct polygons on a geoboard and find the perimeter of each polygon. Students can discuss the strategies used and compare these strategies to finding the area of polygons in the activity above. Have students discuss the differences between the units used to measure area and those used to measure perimeter.

Have students brainstorm and record a long list of situations where finding the perimeter is necessary. Examples: replacing the baseboard in the classroom, building a picture frame, purchasing lace to trim the edges of a tablecloth or fringe to sew around a pillow. Post the list and add to it as new ideas occur.

(C) use the concept of perimeter to solve problems;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify and describe situations where finding perimeter would be useful.	Have students choose perimeter situations from the list above and write original problems to go with the situations. Example: Jeff's club designed a new flag and wanted to fly it above the clubhouse. Jeff made the flag from a piece of fabric that was 3 feet by 2 feet. How much fringe should he buy to trim the edges of the new flag?
Describe the process of finding perimeter.	Use the "Rectangular Garden" activity from TEA Math Module 20, Grades 3-6, Calculator. Have students discuss different solution methods for the activities above and develop a class strategy for finding perimeter. Students can compare it to the class strategy for finding area.

(D) measure the capacity/volume of a container using nonstandard units;

	Sample Clarifying Activities
	Have students find the capacity of a cereal bowl by finding how many spoonsful of cereal it can hold. Students can discuss the "unit" of measure used and how it differs from the units used to measure area and perimeter above. Use activities from Part D in "Operations and Computation." Have students bring in a variety of containers from home and make a list of invented units students could use to find the capacity/volume of each container. Example: To measure the capacity of a soup can, units might include sips of liquid, teaspoons of liquid, cubes, marbles, scoops of rice. Students can discuss the attributes of each of the units and make a decision about each of the units as to its appropriateness in measuring the capacity of the soup can. After estimating first, students can find the number of drops in a spoon using an eyedropper. After estimating first, students can determine how many spoonsful of water are needed to fill an empty soft drink can.

After estimating first, students can use soft drink cans to pour water into a plastic soft drink bottle and determine how many cans are equivalent to a bottle. Have students compare their results to the fluid ounces labeled on the can and bottle.

Have students estimate how many drops of water would fill the plastic soft drink bottle and discuss ways to find out.

(E) identify concrete models that approximate capactiy units;

Sample Learning Objectives

Identify concrete models that approximate one liter (one cubic decimeter).

Identify concrete models that approximate one milliliter (one cubic centimeter).

Identify concrete models that approximate one cup, one quart, and one gallon.

Sample Clarifying Activities

Display a standard measure such as a liter and ask students to make capacity predictions by grouping a variety of containers into categories: containers that hold less than a liter, containers that hold more than a liter, containers that hold about a liter. Have groups use water, rice, or sand to test their predictions.

(F) estimate and measure the capacity of a container;

Sample Clarifying Activities

Have students estimate the number of cups in a gallon milk jug, then use a kitchen measuring cup to pour water into the jug to find out. Students can use a permanent marker to mark the level of the water after each cup is added.

Have students estimate the number of milliliters that will fill a 2-liter soft drink bottle and use a 100ml graduated cylinder to pour water into the bottle to find out. Students can use a permanent marker to mark the level of the water after each cylinder is added.

Have students estimate the volume/capacity of the classroom aquarium by measuring its length, width, and height. Students can then measure its capacity by filling it using the marked milk jug.

(G) solve application and nonroutine problems involving length, weight, time, and capacity;

Sample Learning Objectives	*Sample Clarifying Activities*
Determine the appropriate tool of measurement to solve problems involving length, weight/mass, time, and capacity/volume.	Have students estimate the amount of air in their lungs and discuss a variety of possible ways to measure it. One way might be to blow up balloons with one breath, pushing the balloons under the water of a bucket filled to the spout, catching the overflow, and measuring the water in a marked milk jug. Have students try suggestions, analyze, and compare results.
Justify the unit used to solve problems involving lengths, weight, time, and capacity/volume.	Have students measure the length and mass of a burning candle each hour, recording and calculating the changes that have occurred. Students can discuss which tools seem most appropriate and the units of measure used. Have students make a graph of the results. Have students punch a small hole in the bottom of a soft drink bottle and fill it with water. Students can measure and record the weight and amount of water in the bottle each hour as the water drips out and make a graph of the results. Have students do some research on water clocks and how they work.

(H) determine the amount of time elapsed in a problem-solving situation;

	Sample Clarifying Activities
	Have students predict and measure the amounts of time required for a snail to move 10cm on different surfaces. Have students predict and measure the amounts of time for a bean plant to grow to a certain size in different growing situations. Have students predict the amount of time they spend watching TV during a week. Students can keep a record for a week of beginning and ending times of their TV viewing and graph the results.

Have students use a TV log from the newspaper to examine the amounts of time shows are programmed for certain audiences. Students can categorize the listed offerings by audience and determine the amounts of time per week for each audience represented and the time slots in the schedule, then graph the results. Ask questions such as, "What do the results show? If you wanted to purchase advertising time to sell your latest toy invention, when would you purchase time? Why?"

Ask students, "How could you find out the amount of time devoted to advertising during an average Saturday morning?" Students can discuss a variety of strategies, try them out, and compare results.

Ask a travel agent for some examples of travel itineraries. Have students examine them for a variety of time-related situations. For example, if several flights are involved, how much layover is there between flights? Will the traveler be changing terminals or gates within a large airport? Will the layover time be enough to get from one gate to another in time to get a bite to eat? Will checked baggage have enough time to get from one airplane to another? Pick up some airline schedules at the airport or a travel agency and have students plan a trip itinerary.

(6) Geometry. Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) illustrate the reflections, rotations, and translations of geometric figures using concrete models;

Sample Learning Objectives

Identify examples of reflections in real-world situations.

Identify examples of rotations in real-world situations.

Identify examples of translations in real-world situations.

Sample Clarifying Activities

Take students out onto the playground to act out transformations and rotations. Students can use the slide to illustrate translations. Rotations can be demonstrated using the swing or the merry go round, turning cartwheels, or doing military type turns. Two students can act out reflections by pretending to be mirror images of each other.

Have students trace shapes onto pieces of wax paper, then carefully move the wax paper to slide the imprint, rotate it, or reflect it.

Have students collect logos that have a line of symmetry from the phone book, magazines, or catalogues. Students can fold these or use a mirror along the lines of symmetry to illustrate reflections.

Have students use pattern blocks to create a design. Students can transfer the design onto triangular grid paper or use die cuts of pattern blocks to glue the design to paper. A partner can make a translation, rotation, or reflection of the original design.

Have students create shapes on geoboards and transfer the shapes to dot recording paper. Students can select a peg on the geoboard to be the center of the rotation; rotate the board 1/4 turn, 1/2 turn, 3/4 turn around that peg; and make a sketch after each rotation and compare it to the original design.

Have students classify the letters of the alphabet or pattern blocks into groups that can be reflected and/or rotated and remain looking the same. Students can connect these examples to the concepts of line symmetry and rotational symmetry. See activities in TEA Module 7, Grades 3-6, Geometry.

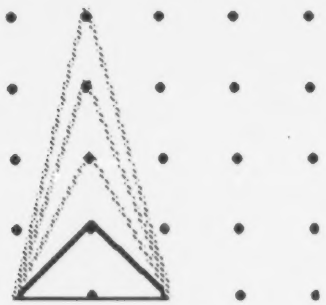
Have students make as long a list as possible of real-world examples of translations, rotations, and reflections. Examples might include: (rotations) a door being opened, a doorknob being turned, a revolving door, a record on a turntable, an electric fan; (translations) an escalator or elevator, a vacuum cleaner being used to clean, a drawer being opened and closed; (reflection) a pancake being flipped, a playing card being turned over, a hamburger being turned.

(B) investigate patterns generated by changing geometric shapes, using models and computer graphics when appropriate;

Sample Clarifying Activities

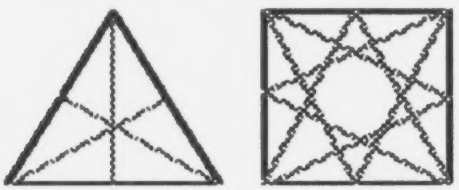
Have students draw a rectangle on graph paper, double the dimensions several times, and record the new dimensions in a table. Students can look for patterns in the perimeters of the rectangles, describe the patterns, and use the patterns to make predictions about the perimeters of the next few rectangles in the sequence.

Have students make an isosceles triangle on a geoboard and find its area if the square enclosed by four pegs represents one unit of area. Students can move the apex one peg at a time and record the resulting areas in a table, look for patterns, describe them, and use the patterns to make predictions about the areas of the next few triangles if the sequence could continue on a large geoboard.



Have students use the software package Geometric pre-Supposer (Sunburst Communications) to explore patterns generated by changing geometric shapes.

Have students start with an equilateral triangle and draw a straight line connecting each vertex to the midpoint of each nonadjacent side. Have them record in a table the line segments drawn from each vertex and the total number of line segments in the figure. Have students try the same thing with a variety of regular polygons (square, pentagon, hexagon, septagon, octagon), record the data in a variety of ways (diagrams, numbers in a table, graphs), and describe any patterns that develop.



	Students can create Logo procedures such as the following, changing the FD number each time: To square, Repeat 4 (FD 30 Rt 90), END. To Rec, Repeat 2 (FD 20 Rt 90 FD 50 Rt 90), END. Have students record the change in the FD number and notice the resulting figure. Do any patterns occur?

(C) compare and contrast angles in relationship to right angles;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify angles that are greater than, less than, or equal to right angles.	Have students trace a right angle using wax or tracing paper and use it to compare to other angles. Students can predict before measuring if an angle is greater than, less than, or about the same as a right angle and categorize the angles by their relationship to a right angle. Have students construct an angle maker by joining two cardboard strips with a brass fastener and experiment making different sized angles. Students can use their angle makers to investigate angles found in real-life objects or situations: the hands on a clock at different times, the corner of a room, an open pair of scissors, corners of books, legs on a table. Have students make a chart, grouping each example according to its relationship to a right angle (<, >, or = a right angle).
Look for patterns of how many right angles can be in certain kinds of two-dimensional figures.	Have students investigate two- and three-dimensional geometric shapes with their angle makers and group objects according to the sizes of their angles. What conclusions can be drawn? Have students go on an angle hunt around the school. Students can list examples according to three categories: <, >, or = a right angle. Ask questions such as, "Can you find examples to fit each category? Which category seems to have the most? Try to explain why."

(D) identify and construct models of intersecting lines, parallel lines, perpendicular lines, right angles, and related two- and three-dimensional figures;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete objects to demonstrate understanding of intersecting lines, parallel lines, perpendicular lines, and right angles.	Have students construct a variety of models of two-dimensional geometric figures: triangle, square, parallelogram, rectangle, trapezoid, hexagon, and so on and classify the figures according to the relationships of their sides.
Explore various combinations of these concepts to see what kinds of two and three-dimensional figures can be constructed.	Have students play Guess My Rule. Each person in the class constructs an interesting shape on a geoboard. Make up a secret rule, such as figures with at least one pair of parallel sides; have all students whose shapes follow the secret rule stand while the others remain seated. The class will try to analyze what is common in the shapes that follow the secret rule, suggest what the rule might be, look for counterexamples among students seated, and eventually identify the rule.
	Have students create models of angles with toothpicks as rays and gumdrops or marshmallows as vertices. These models can be joined to make models of various polygons and polyhedra (two- and three-dimensional figures).
	Have students look for parallel and perpendicular lines in the letters of the alphabet and make organized lists of letters that contain each. Students can do research on designing fonts.
	Have students look through pictures in magazines for real-life examples of parallel lines, intersecting lines, perpendicular lines, and right angles.
	Have students collect examples of optical illusions that contain parallel lines and perpendicular lines and discuss the importance of definitions as opposed to observations in determining these relationships.
	Have students construct three-dimensional figures using straws and clay. Students can build structures that possess various types of lines and angles and use these to design a playground or space station.

Have students fold paper models of three-dimensional figures and explore the numbers and sizes of angles, faces, and edges of each one.

(7) Probability, Statistics, and Graphing. Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) collect, record, and organize data into tables, charts, bar graphs, and line graphs;

Sample Learning Objectives

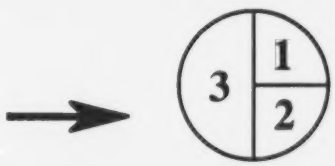
Based on survey data collected, determine the most appropriate way to organize information and display data in a table, chart, or graph.

Collect experimental data and organize the information in an appropriate graph.

Sample Clarifying Activities

Have students roll two dice and record on a chart the sums that come up. Students can organize the data and display these in a bar graph.

Have students use a spinner such as this one to collect data on the following situation:.



If two people spin and add the numbers that come up, which sum will occur most often? Students can organize the data and display these in a bar or line graph.

Have students plant a pinto bean, measure its growth each day, and represent the data in a line graph.

Survey the class concerning right- or left-handedness. Students can organize the data and display these in a bar graph.

Have students compare several brands of paper towels in terms of some measurable characteristic such as absorbency or strength. Students can decide how to collect the data, organize the data, and display the information in a way that supports an argument concerning which paper towel to purchase.

Have students bring a variety of plastic containers into the classroom and categorize them by recycling number. Students can display the information in graph form.

(B) formulate questions and make predictions based on organized data;

Sample Learning Objectives	*Sample Clarifying Activities*
Study a variety of graphs and generate questions that can be answered from the graphs and questions that cannot be answered from the graphs.	Using the graph from Part A in Probability, Statistics, and Graphing, have students study the pinto bean graphs and write questions that can be answered from the graphs and questions that cannot be answered from the graphs.
Use data from a survey to make decisions.	Have students use the information from the class handedness graph to predict the number of right-handed students and the number of left-handed students in the entire school. Students can collect more data to test the prediction.
Use data from an experiment to make predictions.	Have students use the information from the plastic recycling graph to predict which kinds of plastic can be expected in a recycling center that accepts plastic. Students can do some research about which companies purchase each kind of recyclable plastic.

(C) plot points on a coordinate plane that represent ordered pairs of whole numbers arising from problem situations;

Sample Learning Objectives	*Sample Clarifying Activities*
	Use activities from Part B in "Patterns, Relations, and Functions."
Make a table or chart representing the relationship of two sets of data.	Have students make a table of CDs and cost: 1 CD costs $12.00, 2 CD's cost $24.00, etc. Students can record the data from the table as ordered pairs and plot the points on a graph that represent the relationship of CD's to cost.
Translate the information in the table or chart into ordered pairs.	Have students use a balance scale to find out how many toothpicks will balance one popsicle stick. How many toothpicks will balance two popsicle sticks? Three? Students can continue collecting data and record as ordered pairs in a chart. Have students predict how the graph will look before the points are plotted, then try it. Students can use information from the graph to predict how many toothpicks it will take to balance ten popsicle sticks.
Use the ordered pairs to represent the relationship on a graph.	Have students do the first activity in Part B of "Patterns, Relations, and Functions." Students can then predict how a graph of the ordered pairs will look, then plot the points to make the graph.

(D) solve application and nonroutine problems for situations involving graphs;

Sample Learning Objectives	*Sample Clarifying Activities*
Answer the questions generated from class graphs.	Use the graphs from Part A. Have students answer the questions that were generated about the pinto bean graph.
Using the information generated from experimental data, make predictions to solve problems.	Have students use the information generated from rolling two dice to design a game where the strategy is based upon the roll of two dice.
Make decisions based upon information generated from class graphs.	Have students use the information generated from the spinner graph to design a spinner and a game in which each person has an equal chance of winning.
	Have students use the information on the class handedness graph to make decisions concerning how many left- or right-handed fielder's mitts should be purchased for sale in a sporting goods store. What other variables might affect the data?

(E) explore proportions by reducing or enlarging drawings using grids;

Sample Clarifying Activities

Have students draw a one cm grid on a small magazine picture of their favorite electric guitar. Students can use the grid to enlarge the picture to a size that would fit on a poster for the wall of a room. Have students use the idea of proportion to determine what size the second grid will need to be.

Have students draw around a partner's body on butcher paper then draw a grid on top of the outlined body. Students can use the grid to make a "half-sized" person on another piece of butcher paper. Ask questions such as, "How will your grid need to change in size in order to fit your outline on a piece of 8 1/2 x 11 typing paper? Try it and see."

(F) list the possible outcomes in a given situation;

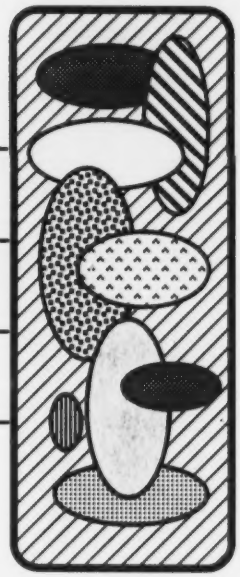
Sample Learning Objectives	*Sample Clarifying Activities*
List the possible outcomes of a single-stage experiment.	Give students a spinner and have them list the possible outcomes of spinning it once (a single-stage experiment).

List the possible outcomes of a multi-stage experiment.	Have students consider throwing two dice (or one die twice) (a multi-stage experiment), and list all the pairs of addends for each of the sums of the two dice. Why are (3,2) and (2,3) considered different outcomes?
Compare and contrast lists of outcomes of different experiments.	Have students compare the possible outcomes of drawing two chips out of a bag containing one blue, one red, one yellow to drawing two chips out of a bag containing two of each of the colors. Students can compare the possible outcomes of drawing both chips at the same time to drawing the second chip after putting the first chip back in the bag.

(G) display all arrangements of a set of objects and identify the pattern;

Sample Learning Objectives	*Sample Clarifying Activities*
Make a list of some of the ways that a set of objects can be arranged. Organize the list and describe a pattern.	Have students list all the ways that four books can be arranged on a shelf, organize the list, and look for a pattern.
Set the pattern to find more ways the set of objects can be arranged. Develop an organized approach to determine if all possible arrangements have been found.	Have students determine how many different ways Jennifer, David and Isaac could be assigned the jobs of Chairman, Treasurer, and Secretary on a committee. Students can display their results in an organized list and use the patterns to predict the ways for four people to be assigned to four different jobs. Felicia has three shirts (red, green, yellow) three pairs of pants (black, blue, brown) and two hats (straw sun hat, baseball cap). How many different outfits can she put together? Have students organize the search and explore different ways to look for patterns. Have students work in groups to address the following problem: "Your friend has five videotapes you want to borrow, but you can only borrow two at one time. How many possible pairs are there?" Groups can present their results to the rest of the class and point out important aspects of the problem to consider, such as whether borrowing videotapes A and B is different from borrowing B and A.

Texas Assessment of Academic Skills



Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Translate whole numbers (name to numeral/numeral to name)
(b) Compare and order whole numbers
(c) Use whole number place value
(d) Round whole numbers (to nearest ten or hundred)
(e) Recognize decimal place value (tenths and hundredths; using models)
(f) Use odds, evens, and skip counting
(g) Recognize and compare fractions using patterns and pictorial models

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use whole number properties and inverse operations
(b) Determine missing elements in patterns
(c) Use number line representations for whole numbers and decimals

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize two- and three-dimensional figures and their properties
(b) Identify informal representations of congruence and symmetry

Objective 4: The students will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Solve problems with metric and customary units and problems involving time (simple time conversions; elapsed time)
(b) Find perimeter
(c) Find area (with grids)

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Determine possible outcomes in a given situation
(b) Analyze data and interpret graphs (including line graphs)

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers and decimals (tenths and hundredths; using models)

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract whole numbers and decimals (tenths and hundredths; using models)

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Multiply whole numbers

Objective 9: The student will use the operation of division to solve problems.

(a) Divide whole numbers (using multiplication facts)

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate with whole numbers

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Select strategies or solve problems using basic operations with whole numbers
(b) Determine strategies or solve problems requiring the use of geometric concepts

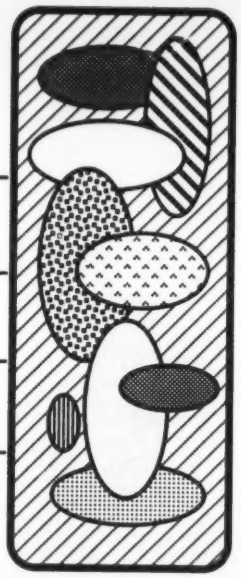
Objective 12: The student will express or solve problems using mathematical representation.

(a) Formulate solution sentences
(b) Interpret charts, picture graphs, and bar graphs and use the information derived to solve problems

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Sample Lessons for Teaching Grade 4 Mathematics



The following sample lessons represent the kind of mathematical experiences recommended for students in fourth grade mathematics classes. These expanded sample activities include ideas for motivaional introductions, exploratory questions to ask during activities, summary questions for reflection after exploring the concept, and ideas for extension and assessment. They are included as examples of significant, mathematical tasks that address the state's curriculum requirements in light of the national recommendations. Note that each sample activity involves several essential element descriptors, as well as several objectives from the Texas Assessment of Academic Skills (TAAS). Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable elementary school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics.*

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groups promotes communication and mathematical confidence and enhances students' problem-solving abilities.

EEs: 2B, 2C *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 3C, 4B, 4F, 5G, 7A, 7B* *TAAS Objectives: 1, 2, 4, 8, 9, 10, 11, 12*

Objective The student will record the pairing of members of two sets, determine the relationship between the members in the pairs, and use the rule to generate additional ordered pairs.

Activity Am I Related to Myself?

Materials Calculators; string; rulers or measuring tapes (metric and customary)

Procedure

Introduction:

1. Have each student use a measuring tape or string and a ruler to measure the distance around his or her head. Have each student also measure his or her own height. The group should then organize these two groups of information in a chart or table to present to the rest of the class, along with two or three summarizing statements about the information in the chart or table.
2. With the whole class, make a table comparing head measurements to height measurements. From the information in the whole-class table, and after discussion of the summarizing statements made by the groups about the possible relationships illustrated in the table, students should try to predict an estimate of the distance around the teacher's head, given his or her height. Measure to check the prediction.

Exploration:

Back in their small groups, students should investigate other possible relationships between different body measurements (e.g., around head to around wrist, length of foot to length of forearm). Each relationship should be supported with a table of ordered pairs and summary statements. Some questions for students to consider as they are exploring are:

- How does the table of ordered pairs help you look for a relationship?
- Are you able to find an exact relationship between the members of the ordered pairs, or is it more of an approximate relationship? Why do you think this is so?
- How many ordered pairs do you feel like you need to generate before you can begin to look for a relationship?
- Do you think there will always be a relationship between the members of a set of ordered pairs, no matter what the data represent?

Extension:

- Write an equation to describe the relationship you found in a table of ordered pairs.
- Generate some stories about how knowing these relationships might be useful.
- Graph other relationships from your tables of ordered pairs and compare the different graphs. Also make collections of other sets of related information (e.g., grams of fat compared to calories of fat, length of phone call compared to cost of phone call) gathered from magazines, newspapers, and world almanacs.

Summary:

Let members of each group share their strategies for organizing their information and identifying relationships between different body measurements. To follow up on the questions asked during the exploration, have students discuss the following questions:

- What effect does the approximate nature of measurement have on your data and on the statements you make about the relationships you observe?
- What are the benefits of organizing the information in ordered pairs within a table?

- What data did you gather that did not seem to be described with a particular relationship?
- For those of you who tried to write equations to describe your relationships, how do they relate to the summary statements made by you and members of the other groups.
- Are there any other factors we might need to consider along with the statements of our relationships? For example, are these relationships true for humans of any age?

During the summary, the class might choose one set of ordered pairs to graph on a coordinate grid to produce a visual representation of a particular relationship.

Assessment

- teacher observation of student participation during exploration and discussion
- group-generated tables of organized ordered pairs
- summary statements describing the observed relationships in the tables of ordered pairs
- use of summary statements to predict possible additional ordered pairs in the relationship

EEs: 3A, 3B *Related EEs: 1A, 1B, 1C, 2A, 4E, 4H, 7A, 7B* *TAAS Objectives: 1, 2, 8, 11*

Objective The student will use concrete materials to represent factor pairs of numbers. The student will use the patterns of factor pairs to identify prime and composite numbers.

Activity Multiple Towers

Materials Interlocking cubes; hundreds charts; "Multiple Towers" worksheet

Resources *The Math Solution* by Marilyn Burns.

Procedure

Introduction:

1. Have the students build towers of cubes on a hundreds chart. Explain that these towers will be built using the directions on the Multiple Towers worksheet.
2. Discuss the first step: Put a blue cube on multiples of 2. Ask, "How will you decide on which numbers to put the cubes? Who can explain what we mean by a multiple? Can you give other examples?" Then place the cubes on the appropriate numbers.

Exploration:

Let partners work together following the worksheet directions and answering the questions on the sheet. Circulate around the room, discussing with pairs of students these questions:

- What is your strategy for finding multiples?
- Can you identify a pattern to help you?
- Do you think every number will have a tower?

As pairs finish their towers and questions, ask them to design a recording system to show which cubes ended up on each number, then record their findings.

Extension:

- Write your own questions concerning observations you have made about the towers. Trade questions with other students and answer theirs.
- Write statements concerning prime numbers, composite numbers, or patterns of factors.
- Design another recording system for your results.
- Build towers for numbers greater than 100.
- Predict which numbers would have tall or short towers.

Summary:

When all groups have completed the activity and have recorded their results, pose these questions to the class:

- How did you decide where to put each color of cube?
- Which numbers had the tallest towers? Why? Which colors were in them?
- Which numbers had no cubes? Why?
- Which numbers had only one cube? Why?
- Did you notice that when you put a black cube on a number, it already had a green cube on it? Why do you suppose that happened?
- What happened when you removed the towers and tried to put them back on the correct numbers? What did you do to try to figure out where they belonged? Did anyone try anything different? How did it work?
- Did you discover anything else from the activity that we have not already discussed?

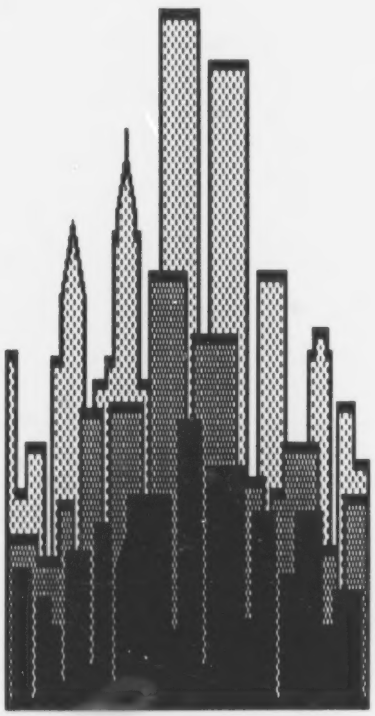
Assessment

- Teacher observation of student participation during exploration and discussion
- Students' records of their findings
- Oral or written statements of observations or patterns discovered during the activity

MULTIPLE TOWERS

You need: Unifix cubes, 100s chart, a partner

Put a blue cube on multiples of **2**.
Put a green cube on multiples of **3**.
Put a yellow cube on multiples of **4**.
Put a red cube on all multiples of **5**.
Put a brown cube on multiples of **6**.
Put a white cube on multiples of **7**.
Put an orange cube on multiples of **8**.
Put a black cube on multiples of **9**.
Put a maroon cube on multiples of **10**.



- **Which numbers have the tallest towers?**
- **Which numbers have no cubes?**
- **Which have only one cube?**
- **Now remove the towers. (Be careful not to break any.)**
- **Try to put them back on the correct numbers.**
- **Write about how you did this.**

EE: 4H *Related EEs: 1A, 1B, 1C, 4A* *TAAS Objectives: 1, 6, 7, 10, 11, 12, 13*

Objective The student will use properties of operations and problem-solving strategies to do mental calculations, extending beyond fact recall.

Activity Hooked on Problem Solving

Materials Number tiles with the digits 1-9 (or numbered squares of paper); 5 x 7 index cards; markers; tape

Resources "Hook Your Students On Problem Solving" by Gloria J. Bledsoe in *Arithmetic Teacher*, December 1989.

Procedure

Introduction:

1. Have students arrange their numbered squares into a 3 x 3 array. Ask, "Does the sum of the first two 3-digit numbers equal the third 3-digit number? (Probably not!) Is it possible to arrange the digits so that the sum of the first two numbers does equal the third 3-digit number?"
2. Have students try to create such an arrangement. Hint: Regrouping is an important consideration. For example:

$$\begin{array}{r} 291 \\ +\,384 \\ \hline 675 \end{array}$$

Exploration:

- As students begin to find solutions, have them record the solutions on 5 x 7 index cards with heavy felt-tip markers. Post the solutions. This gives students a chance to notice duplicates, put the cards in categories, and begin to search for patterns.
- Once a fairly large collection of data is posted, ask, "How many solutions do you think might be possible? Can you find a pattern that might make the task easier?"
- Record the patterns noticed as "Conjectures" and ask students to search for counterexamples to test their conjectures. Conjectures that can be logically explained or for which no counterexample can be found can be listed under "Generalizations." Possible examples of conjectures:
  - It is always necessary to regroup to find a solution.
  - Neither a 1 nor a 2 can be the hundreds digit of the answer.
  - The digits in the answers add up to 18. (This pattern is difficult to notice at first. To speed up the process, ask students to concentrate on the sums.)

Extension:

- Write a summary statement explaining how to solve the problem.
- Is it possible to arrange the digits in some way to create problems using operations other than addition? Experiment and record your findings.

Summary:

- Explain how you arrived at your conjectures and generalizations.
- Can anyone find a counterexample we haven't thought of?
- How many possible solutions did you find?
- How did you use your patterns and conjectures to find more solutions?
- What strategies did you use to verify that you had found all the solutions?

Assessment

- teacher observation of student participation in exploration and discussion
- students' explanations of strategies to determine patterns
- students' identification and justification of patterns that led to conjectures and generalizations
- students' use of conjectures and generalizations to find new solutions
- student-generated counterexamples to disprove false conjectures.

EE: 5G *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 5D, 5E, 5F, 5H, 7B* *TAAS Objectives: 4, 10, 11, 12, 13*

Objective The student will explore ways to quantify and record the strengths of various magnets in order to compare and evaluate them.

Activity Magnets—How Strong Are They?

Materials A variety of magnets of different shapes and sizes; a variety of measuring devices such as balance scales and masses, tape measures, graduated cylinders, rulers, spring scales, measuring cups and spoons, stopwatches for use with both customary and metric units.

Procedure

Introduction:

1. Use iron filings, or a magnetic field projector, to display the magnetic field of a magnet. Have students describe the patterns they see.
2. Display the magnetic field of another magnet or of the same magnet after remagnetizing it. Have students discuss the differences they see.
3. Have students brainstorm ways they could measure strengths of magnets in order to compare them.
4. Have each group of students devise a way to measure the strengths of two magnets, make and record their measurements, and present their results to the class.
5. Discuss the various methods of measurement used and the groups' results.

Exploration:

- Can different magnets have different strengths? Can the same magnet have different strengths?
- What measuring tools could you use to measure the magnet's strength?
- How could you use (a particular measuring tool) to measure the magnet's strength?
- What kinds of objects do you have to use along with the measurement tools? What will the magnet attract?
- If you test a magnet over again, do you get exactly the same result? Why or why not?
- After testing a magnet once to measure its strength, did you make any changes to your method of measurement? Why or why not?
- Can you use the repelling characteristic of the magnets to measure strength? If so, how? If not, why not?
- How did your group decide to record your data?
- What conclusions can you make from your data?
- Does the size of the magnet affect the strength? How do you know?
- Does the shape of the magnet affect the strength? How do you know?
- Do two magnets that look exactly the same have the same strength? How do you know?

Extension:

- Design a display of your results (a poster, table set-up, etc.).
- Design another way to measure your magnets' strengths. Compare the results to your first set.
- Have the teacher remagnitize your magnets, measure their new strengths, and compare to their old strengths. Did their strengths really increase? How do you know?

Summary:

- What techniques did your group use to measure the strength of the magnets?
- How did your group decide what to use to measure?

- How did your group decide how to represent your data?
- What conclusions did your group make from your findings?
- How were your group's results different from or the same as other groups' results?
- What do you think caused the differences?
- What if you had used magnets of different sizes, shapes, . . .?
- How could we determine which was the strongest magnet? The weakest one? Do we have enough information to determine that now?
- How or when could this information be useful to you?
- If you did this again, would you use your same technique again or that of another group? Why?
- How might you be able to test the strength of a magnet that was hidden from view? Could you use some of your same techniques?
- How did mathematics help you in your exploration of the strength of magnets?

Assessment

Questions:

(See summary questions.)

Observations:

- Were students actively participating?
- Were students making measurements appropriately?
- Were students making appropriate suggestions for ways to measure magnet strength?
- Were students using appropriate instruments for each type of measurement?
- Were students using knowledge about measurement units?

Tasks:

- In your journal, describe how your group decided to measure magnet strength and why you decided to measure it this way.
- Write a story about a situation in which these data would be useful to someone.

EE: 5E *Related EEs: 1A, 1B, 1C, 1D, 7C, 7D* *TAAS Objectives: 2, 3, 4, 8, 9, 10, 11, 12, 13*

Objective The student will measure and compare body height and jump height in order to determine a relationship between the two. This will be called the springy legs factor.

Activity Springy Legs

Materials Calculators; four colors of felt-tip markers for each group; tape measures; string; long paper; large paper for graphs and data tables

Resources Books about Michael Jordan or basketball

Procedure

Introduction:

Discuss what makes Michael Jordan a good basketball player. Show statistics of Michael Jordan (height 78" and jump height 48"). Discuss how high he can jump. Students can demonstrate how high 48 inches would be. Discuss how you can express Michael's jump as a portion of his height. It will be called the "Springy Legs Factor" (SLF). It is determined by: jump height ÷ body height x 100. Michael's SLF would be 48 ÷ 78 x 100 = 61.5%. He can jump more than half his height! Ask students, "What part of your height can you jump?" Demonstrate the procedure for how students will compute their own SLF. Divide students into teams of four. Each member of the teams will complete the following steps (as teacher demonstrates):

1. Hold the marker in one hand and stand close to the wall, facing a long piece of paper taped to the wall. With your feet flat on the floor, reach up and make a mark on the paper as high as you can.
2. Still facing the wall, jump straight up and make another mark as high as you can.
3. Measure the distance in inches between the two marks. This is your jump height (JH). Record it in the data table.
4. Next, measure your body height (BH)—head to toe—and convert it to inches. Record it. (The JH and BH for each person can be recorded in an ordered pair, (BH, JH).
5. Divide your JH by your BH and multiply the answer by 100. This is your Springy Legs Factor (SLF). Record it.

Exploration:

- Are you holding your marker the same way each time you mark the paper? Will it make a difference in your results?
- Who do you think can jump the highest? Will that predict which student has the greatest SLF?
- Do the tallest people have the greatest SLF?
- Once you measured BH and JH, can you predict who will have the highest and lowest SLF?

Extension:

- Cut a piece of string the length of your BH and the length of your JH. Compare the two. Is your JH more than half of your BH like Michael Jordan's?
- Out of your four group members, whose jump height is a greater portion of their BH? Rank all four members' strings using this comparison (lowest to highest).
- Rank your group according to SLF from lowest to highest. Are the rankings similar?
- Suppose you grew to be as tall as Michael Jordan (78") but kept your same SLF. How high would you be able to jump?

- Collect data on other famous basketball players. Predict their SLFs. How do they compare to Michael Jordan's?
- Can you find a way to record a relationship between JH, BH, and SLF? (Hint: use ordered pairs and graphs.)
- Do sizes of other body parts have an effect of the SLF? (consider arm length, foot length, and leg length.) Discuss in groups and be prepared to make a statement.

Summary:

- Is it possible to predict, by looking at your classmates, who will have the highest SLF? Why or why not? If yes, what factors influenced your decision?
- Collect class data and make a class graph. Were your predictions correct? Does the tallest person have the highest SLF? Does the shortest person have the lowest SLF?
- What determines the SLF? (Percentage of jump height to body height. Persons who can jump a greater part of their body height will have higher SLFs than persons who jump a smaller proportion to their own body height.)
- Do persons who have the same jump height have the same SLF?
- Using the tallest person and the person with the highest SLF, cut strings to show their body height and jump height and compare the two. Whose jump height is a greater portion of their body height?
- To get a sense of the relationship among variables, have the students line up in order of: Body height (tallest to shortest), Jump height (highest to lowest), SLF (highest to lowest). Discuss results.
- Plot data on coordinate graphs that compare body height and jump height, body height and SLF, and jump height and SLF. Interpret by eyeballing a straight line through the data points.

Assessment

Questions:

- Choose a winning basketball team from our class. What criteria did you use?
- (See also summary questions.)

Observations:

- How successful were the students with calculators? Did the calculators hinder or help work?
- Do students need additional practice measuring?
- Can students compile and interpret results from a graph?
- Did students accurately measure BH and JH?
- Did each group record data according to instructions?
- Were the students able to predict who had the highest SLF?
- Could the students determine a relationship between the BH and JH?
- Could students verbalize relationships among the variables?

Tasks:

- Give students the JH and BH of two different students. Predict who would have the highest SLF. Calculate.
- Make a true statement about information on the class graph.
- In journals, write what you have discovered about the Springy Legs Factor. Tell how it is determined and what influences it.

EEs: 6B, 6D *Related EEs: 1A, 1B, 1C, 6A, 7A* *TAAS Objectives: 3, 11*

Objective The student will explore polygons, identifying their attributes and practicing vocabulary, by manipulating tangram pieces.

Activity Tantalizing Tangrams

Materials 9" x 9" construction paper squares (several for each student); scissors; tangram chart (one per group); box; variety of objects

Procedure

Introduction:

1. Give students a box and a variety of objects of different shapes and sizes. Have students try to pack the objects into the box so they fit perfectly to prevent breakage during shipping. Discuss different strategies. (Could be done as a center the week before the tangram activity.)
2. Tell the legend of the tangram as a flannel board story.
3. Demonstrate how to make the tangram and have students follow along with you. Use appropriate terminology as the tangram pieces are created (isosceles right triangle, congruent, similar, etc.).
4. Compare attributes of various pieces.
5. Generate a specific name or description for each tangram piece to aid in communication within and between groups.
6. Discuss definitions and characteristics of shapes listed on the tangram chart. For example, how many pairs of parallel sides does a trapezoid have? Why is it important for us to decide on one particular definition of trapezoid?
7. Have students work in small groups to find the given shapes with the given number of pieces to complete the tangram chart.

Exploration:

- Could you flip or turn or replace pieces to make a new shape?
- Could you make that shape another way?
- Are you using any strategies? Describe them (for example, building a new shape from an old one).
- Could you replace one piece in a shape with two pieces? Two pieces with one piece?
- Which piece seems to be the easiest to use? The hardest? Why?
- Which pieces do you seem to use most often? Least often? Why?
- How do you know that what you have made is a (square, trapezoid, etc.)?

Extension:

- Find all the ways to make each shape with a given number of pieces.
- Develop an argument for why there is no six-piece square.
- Design a display where all the solutions could be posted (perhaps one poster for each shape).

Summary:

- What kinds of triangles did you make?
- What are some other ways to construct each of the shapes?
- What strategies did your group use for finding shapes?
- How did you decide if your shape was a square, trapezoid, etc.?
- How did the definitions of the shapes help you?
- Which pieces did you tend to use more than others? Why?

Assessment

Questions:

(See summary questions.)

Observations:

- Were students exhibiting an understanding of the different shapes?
- Were students using appropriate vocabulary?
- Were students looking for patterns and using strategies?
- Were students organizing their data? Did they have an organized system for approaching the chart? Could they explain their systems?
- Were students persistent in looking for shapes?

Tasks:

- Compile a class glossary.
- Make a class poster display.
- Write summary statements or a paragraph in your journal about what you learned about tangrams.

TANGRAM GRID

Use this number of pieces to form a: →	SQUARE	RECTANGLE (that is not a square)	TRIANGLE	TRAPEZOID	PARALLELO-GRAM (that is not a rectangle)
1					
2					
3					
4					
5					
6					
7					

EEs: 7A, 7B *Related EEs: 1A, 1B, 1C* *TAAS Objectives: 5, 6, 11*

Objective The student will collect, record, and organize data into tables, charts, bar graphs, and line graphs. The student will formulate questions and make predictions based on organized data.

Activity Sums of Two Dice

Materials One pair of dice for each pair of students; number line with the numbers 2 to 12 on it (one per pair); 11 counters per pair; recording sheets; large paper for class graph; markers

Resources *A Collection of Math Lessons, Grades 3-6* by Marilyn Burns

Procedure

Introduction:

Arrange the students in groups of 2. For each pair, set aside:

- 1 pair of dice
- number line with the numbers 2 to 12 on it
- 11 counters
- recording sheets (2-3 per group)

Post a large piece of paper (or erase a large section of the chalkboard) for a graph. Across the top, write "Two-Dice Sums". Down the left side, write the numbers 2 to 12.

1. Begin the lesson by holding up a pair of dice. "If I roll two dice, and add the numbers that come up, what is the smallest sum I could get?" Record 2 on the chalkboard or overhead. "What is the largest sum I could get?" Record 12 on the chalkboard leaving room for the possible sums in between. "What about 3? Is a sum of 3 possible? How could I get it?" Continue with 4, 5, 6, etc. up to 12. "How many possible sums are there when we roll two dice and add the numbers that come up?" Verify from the list on the chalkboard that there are 11 possible sums.
2. To begin, teach the game to the whole class. "I'm going to teach you a dice game that you will play with a partner. I'm going to give you and your partner a number line with the numbers from 2 to 12 on it and eleven counters. You need to arrange your counters on your number line. You can do this any way you like. You may put one counter on each number. You may stack all the counters on one number. Or you can group them in any other way you like.

"Once all teams have placed their counters, I will begin to roll the dice. When I roll the dice, I will call out the sum that comes up. If you have a counter on that number, you remove it. For example, if I roll an 8 and you have a counter on 8, you would take it off. If you should happen to have two or more counters on 8, you would take off only one of those counters.

"The idea is to be the first team to remove all of your counters. With your partner, talk about how you would like to arrange your counters to try to be the first team to take them all off."

3. When the teams have had a chance to discuss their strategies and place their counters, play the game.

Exploration:
During the game, notice when students have removed most of their counters and are waiting for only 2 or 3 numbers on their number lines. You may wish to ask such questions as:

- Which numbers are most of you waiting for?
- Is everyone waiting for 2, 11, or 12?
- What sums did I seem to call most often?

"Now that you have had a chance to play a game, it might be interesting to gather some data which may help us play the game better next time." Explain to the students that each pair is going to roll the dice, but this time use the recording sheet on the following page to record with an X each sum that comes up. When one sum reaches the "finish line", record that sum with a tally mark on the class graph. "You may use as many recording sheets as you have time to do."

Extension:

- Play the game again and test the arrangement you described.
- How did your results compare with your prediction?
- How can you explain your results?"
- How would your results change if you rolled one die instead of two dice and recorded the number that comes up? Predict first, then try it and see.
- What would happen if you used polyhedral dice? Make a prediction. Try it and see.
- How do you think your results would change if you rolled three dice instead of two? Predict first, then try the experiment. How many times should you plan to roll the dice to get some meaningful results? Record your results and compare with your prediction.

Summary:
When most pairs have had a chance to complete 2 or 3 recording sheets, begin a class discussion. "Lets analyze our class graph." Questions for the class might include:

- Which sum came up the most often?
- Why do you think this happened? Was it just luck?
- How many possible outcomes are there?
- How many ways can you get a sum of "6"?
  How many ways are there to roll a seven?
- How many ways are there to roll a 2? a 12? What are they?
- What other sums came up on our class graph? How many ways can you get those sums? (Students will try to tell you that you can roll 9 with a 7 and a 2 and an 8 and a 1. Remind them that they are limited by the numbers on the dice.)
- Which sums do not appear on our class graph? Why do you think?
- If there are five ways to get a 6, and we have 36 possible outcomes, how can we record this information? What are some of the different forms in which this information could be recorded?
- Why is it important to record this information in a meaningful way?
- Would someone from another school be able to understand your explanation?
- Why might one way to record be clearer than another?

"If we were to play the game again, think about how you would arrange your counters to try to win the game. Talk it over with your partner. Write a description of the arrangement you would use and explain why you made those choices."

Assessment

Questions:

- Given a description of an activity using 4-sided dice numbered 1-4, have students predict possible and expected outcomes and justify their answers.
- (See also summary questions.)

Observations:

- Did student discussion indicate an understanding of the problem?
- Were students using an appropriate and clear method to record their data?
- Were students arranging their data to help them find patterns?
- Were students using appropriate vocabulary to explain their results?
- Were students discussing the meaning of the patterns they found?

Tasks:

- Make a chart or graph of an activity using a multi-sided die with something other than 6 sides.
- Write suggestions to improve the charts or graphs.
- Explain how you used the chart or graph to design your strategy for playing the game.

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

This chart is intended for example only. Lead the children to develop a chart similar to this after compiling information. It could also include representations of the addends with ordered pairs:

	1	2	3	.	.	.
1	(1,1)	(1,2)	(1,3)	.	.	.
1	(2,1)	(2,2)	(2,3)	.	.	.
1	(3,1)	(3,2)	(3,3)	.	.	.
.						
.						
.						

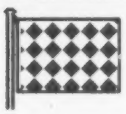
Names: _____

TWO-DICE SUMS

constructed from **A Collection of Math Lessons from Grades 3-6 by M. Burns**

Roll 2 dice. Mark an X under the sum. Do this until one sum reaches the Finish Line.

2	3	4	5	6	7	8	9	10	11	12



Finish Line

Record the sum that finished first on the class chart.

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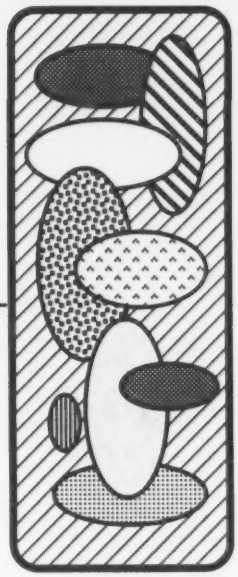
Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems—must be assessed. A student's disposition toward mathematics (confidence, interest, perseverance, etc.) is also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

In summary, the following aspects of students assessment and program evaluation should receive increased and decreased attention (NCTM, 1989):

Increased Attention	Decreased Attention
• Assessing what students know and how they think about mathematics	• Assessing what students do not know
• Having assessment be an integral part of teaching	• Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
• Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	• Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
• Developing problem situations that require the applications of a number of mathematical ideas	• Using exercises or word problems requiring only one or two skills
• Using multiple assessment techniques, including written, oral, and demonstration formats	• Using only written tests
• Using calculators, computers, and manipulatives in assessment	• Excluding calculators, computers, and manipulatives from the assessment process
• Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction	• Evaluating the program only on the basis of test scores
• Using standardized achievement tests as only one of many indicators of program outcomes	• Using standardized achievement tests as the only indicator of program outcomes

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Assessment instruments and techniques should be properly aligned with the curriculum to enable educators to draw conclusions about instructional needs, progress in achieving the goals of the curriculum, and the effectiveness of a mathematics program. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. The particular emphases of the assessment should reflect the emphases of instruction. For example, primary children, whose understanding of fractions is closely tied to the use of physical materials, should be encouraged to use such materials to demonstrate their conceptual knowledge. Assessment items need to be structured around the central ideas of the curriculum and need to provide opportunities for students to demonstrate their understanding of the connections among major concepts. In addition, assessment must reflect the relative emphasis placed on technology during instruction; to the extent that calculators and computers have been important during instruction, they should also be available during assessment.

Assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, and open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. Assessment can occur during and after whole-group explorations, during whole-group discussions, in sharing sessions, during individual conferences, during small-group conferences, while students are working on projects, after completion of projects, when students are engaged in self-evaluation tasks, and continually while students are explaining, justifying, debating, and questioning ideas and concepts.

Using Portfolios. Student portfolios are becoming more prevalent as a means of keeping a record of student progress in mathematics. Teachers have always kept folders of students' work, but portfolios should have more focus and be more important for assessment. An assessment portfolio is a planned selection of a student's work collected throughout the school year. Teachers as well as students should be allowed to choose the items to be included in portfolios, since it gives a good indication of what is valued in the work the students do throughout the school year. A portfolio might include samples of student-produced written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical and graphic representations; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio, and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

Using Writing. Communication in mathematics has become important as we move into an era of a thinking curriculum. Journals, logs, problem-solving notebooks, explanations, justifications, and reflections are ways to include writing in the mathematics curriculum. Students should be urged to discuss ideas with each other, and to ask questions, to diagram and graph problem situations for clarity. Writing in mathematics classes, once rare, is now vital. In particular, mathematics journals can include the following:

- vocabulary definitions written in the student's own words along with explanations of how the terms are used in mathematics
- rules or procedures written as if explained to a friend in a letter or to another student who was absent during the instruction
- free writing, including what students think they will learn in an experience, descriptions of accomplishments, how students can use what they've learned, what isn't fully understood or is causing difficulty, examples in the real world related to the mathematics learned, a discovery made or additional ideas and conjectures related to the topic, and what else students might want to learn about.

These writing experiences are also important mathematics learning experiences in that they:

- help students become more active in their own learning
- help students internalize what they are learning to make it more meaningful
- allow students to express their feelings and attitudes toward mathematics
- give students a source they can use for studying
- allow students to reflect upon and clarify their own thinking
- give students the opportunity to share with each other what they are learning, also allowing them to learn from one another
- allow students to go beyond what they are learning in class and to make conjectures and connections
- give students the opportunity to think of mathematics as existing outside the classroom
- give students the opportunity to communicate with the teacher in an informal setting
- give the teacher an idea of how students are thinking
- allow the teacher to informally assess student learning (whether it be pre- or post-assessment)

Using Teacher Observations. Teacher observations can be broken down into two levels: formal and informal. Formal observations include checklists, comment cards, and summaries. Informal observations include mental notes. Students should be observed both individually and as they work in groups. When using observations, a teacher should look for students' learning styles, students' ideas, communication techniques, cooperation strategies, and use of manipulatives. Some possible questions that can guide observations of students doing mathematics are:

- Does the student consistently work alone or with others?
- Does the student try to explain organizational and mathematical ideas?
- Does the student synthesize and summarize his or her own or a group's thinking?
- Does the student work with the group to agree on a plan or structure for tackling the task?

- Individually or within the group, does the student choose and use appropriate manipulatives?

Using Questioning. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computations are over used and over done. Using good, high-level open-ended questions that give students a chance to think are one of the goals of mathematics assessment. These questions might be used as teaching or leading questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. When using oral questions, the teacher can prepare a list of possible questions ahead of time. (For examples, see the sample activities in the previous section.) The teacher should allow for plenty of wait time. The teacher may keep a written record of observations during the questioning time to use for formal assessment. Questioning for assessment should occur in several places during instruction:

- during introductory activities to assess students' prior knowledge and experience
- during exploration to focus students' attention on important concepts and connections
- after instruction, in order for students to summarize results, reflect on their experience, and clarify their thoughts

Using Student Presentations. Student presentations can take many forms, including oral explanations, oral presentations, and projects. One of the best ways to assure the connection between instruction and assessment is to embed assessment into instruction. When students become involved in projects or investigations, assessment becomes natural and invisible. Student presentations may be related to connections within mathematics and connections outside mathematics. When evaluating student presentations, the teacher should look for whether the student can identify and define the problem; make a plan; collect needed information; organize the information and look for patterns; discuss, review, revise, and explain results; and produce a quality product or report.

Using Performance Assessment. Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take from half an hour to several days to complete or solve. The object of this form of assessment is to look at how students are working, as well as at the completed tasks or products. Performance assessment requires the teacher to look at how students solve a problem. Performance activities may be videotaped, tape recorded, or recorded in writing. The task might be from any mathematical content area and might include some connections such as with science, social studies, language arts, or fine arts. Performance assessment is an excellent place for students to use manipulatives to demonstrate understanding of mathematics content. Information from performance assessment can be recorded using rubrics that assign point values to important aspects of the problem-solving process. For example, the following assessment criteria could be used during observation or based on written work to judge a student's involvement in problem solving:

1. Understanding the Problem:

0 points	Does not understand the problem
1 point	Misunderstands part of the problem
2 points	Completely understands the problem

2. Choosing and Implementing a Solution Strategy

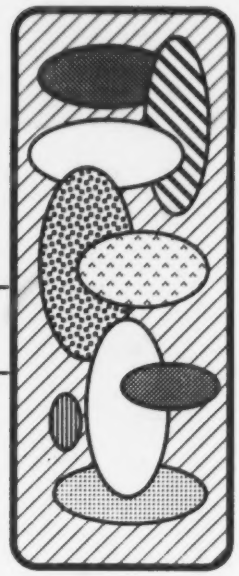
0 points	Makes no attempt or uses a totally inappropriate strategy
1 point	Chooses a partly correct strategy based on interpreting part of the problem incorrectly
2 points	Chooses a strategy that could lead to a correct solution if used without error

3. Getting the Answer

0 points	Gets no answer or a wrong answer based on an inappropriate solution strategy
1 point	Makes a copying error or computational error; gets partial answer for a problem with multiple answers; or labels answer incorrectly
2 points	Gets correct solution

Some excellent resources on assessment, in addition to the NCTM *Curriculum and Evaluation Standards*, include *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (1991) and *Assessment alternatives in Mathematics* (Stenmark, 1989).

References and Resources



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Molyneux, L. (1992). *Cooperative learning, math and success*. Canandaigua, NY: Trellis Books.

National Council of Teachers of Mathematics. (various years). *Arithmetic Teacher* (a journal of philosophy and activities for teachers of elementary mathematics).

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Parker, T. (1984). *In one day*. Boston, MA: Houghton Mifflin.

Pollard, J. (1985). *Building toothpick bridges*. Palo Alto, CA: Dale Seymour Publications.

Popping with power. (1987). Fresno, CA: AIMS Education Foundation.

Stenmark, J. K. (1989). *Assessment alternatives in mathematics*. Berkeley, CA: Lawrence Hall of Science, University of California.

Stenmark, J. K., Thompson, V., and Cossey, R. (1986). *Family math*. Berkeley, CA: Lawrence Hall of Science, University of California.

Wahl, J. and Wahl, S. (1976). *I can count the petals of a flower*. Reston, VA: National Council of Teachers of Mathematics.

Children's Trade Books

Numerous children's books have the potential for motivating rich mathematics activities. This short list gives the bibliographic information of the books mentioned in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5).

Anno, M. (1982). *Anno's counting house*. New York: Philomel Books.

Anno, M. (1983). *Anno's mysterious multiplying jar*. New York: Philomel Books.

Aardema, V. (1976). *Why mosquitoes buzz in people's ears*. New York: Dial Books for Young Readers.

Bemelmans, L. (1960). *Madeline*. New York: Viking Children's Books:

Bishop, C. H. (1938). *Five Chinese brothers*. New York: Coward.

Brier, C. (1983). *The shoemaker and the elves*. New York: Lothrop, Lee & Shepard.

Carle, E. (1977). *The grouchy ladybug*. New York: Scholastic, Inc.

Carle, E. (1989). *The very hungry caterpillar*. New York: Putman.

Dahl, R. (1964). *Charlie and the chocolate factory*. New York: Alfred Knopf.

de Paola, T. (1978). *The popcorn book*. New York: Holiday House.

Ehlert, L. (1990). *Fish eyes: A book you can count on*. San Diego: Harcourt, Brace, Jovanovich.

Faucher, E. (1989). *Honey, I shrunk the kids*. New York: Scholastic, Inc.

Flournoy, V. (1985). *The patchwork quilt*. New York: Dial Books for Young Readers.

Freeman, D. (1968). *Corduroy*. New York: Viking Press.

Giganti, Jr., P. (1992). *Each orange had 8 slices*. New York: The Trumpet Club.

Haley, G. E. (1971). *Noah's ark*. New York: Atheneum.

Hogrigan, N. (1966). *Always room for one more*. New York: Holt.

Hulme, J. N. (1991). *Sea squares*. New York: Hyperion Books.

Hutchins, P. (1987). *The doorbell rang*. New York: Scholastic, Inc.

Lobel, A. (1983). *Frog and toad are friends*. New York: Harper.

Mathews, L. (1978). *Bunches and bunches of bunnies*. New York: Scholastic, Inc.

McMillan, B. (1991). *Eating fractions*. New York: Scholastic, Inc.

Owen, A. (1988). *Annie's one to ten.* New York: Alfred Knopf.

Parish, P. (1963). *Amelia Bedelia.* New York: Harper and Row.

Peterson, J. (1967). *The Littles.* New York: Scholastic, Inc.

Potts, J. (1992). *The house that makes shapes.* New York: Hyperion.

Prelutsky, J. (1984). *New kid on the block.* New York: Greenwillow.

Prelutsky, J. (1978). *The Queen of Eene.* New York: Greenwillow.

Price, L. (1990). *Aida.* New York: Harcourt, Brace, Javonovich.

Sendak, M. (1962). *Chicken soup with rice.* New York: Harper and Row.

Silverstein, S. (1981). *A light in the attic.* New York: Harper and Row.

Silverstein, S. (1974). *Where the sidewalk ends.* New York: Harper and Row.

Singer, M. (1991). *Nine o'clock lullaby.* New York: HarperTrophy.

Steptoe, J. (1980). *Daddy is a monster . . . sometimes.* New York: HarperTrophy.

Ward, C. (1988). *Cookie's week.* New York: Putnam.

West, C. (1987). *Ten little crocodiles.* New York: Barron's.

Yolen, J. (1987). *Owl moon.* New York: Philomel Books.

Software

The following list contains bibliographic information for the software packages mentioned in this series of curriculum guides for elementary mathematics (Grades 1 - 5). Other appropriate software may be obtained from these and other companies.

Blockers and Finders from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Geometric preSupposer from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Hands-On Math: Volumes 1, 2, and 3 from Ventura Educational Systems, 3440 Brokenhill Street, Newbury Park, CA 91320.

Suggested Manipulatives

The following is a list of the manipulative materials used in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5):

Calculators
Base ten blocks
Coins and bills (play or real money)
Interlocking cubes
Colored tiles
Pattern blocks
Cuisenaire rods

Graphing floor mat
Polyhedral dice (including the regular cube)
Colored chips
Two-color counters

Attribute blocks
Geoblocks
Geoboards
Tangrams
Plastic mirrors
Wooden or plastic models of geometric solids

Balance scales and masses (customary and metric)
Spring scales
Tape measures (customary and metric)
Rulers (customary and metric)
Meter sticks and yardsticks
Trundle wheels
Graduated cylinders
Measuring cups and spoons
Stopwatches

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